The Bar Mode Instability: Gravitational Torques and Bar Formation

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Abstract

Dynamic nonaxisymmetric instabilities in rapidly rotating fluids have several potential applications in astrophysics. Here, we investigate the effects of nonaxisymmetric instabilities on the evolution of rapidly rotating polytropes and then apply our results to the star formation process. The most unstable nonaxisymmetric mode is the lowest order mode, the bar mode. Contrary to its name, the bar mode produces barlike distortions only in the central regions of the polytropes; it trails into spiral arms as the equator is approached. The winding gives rise to self-gravitational torques which control the nonlinear evolution of the object. Here, we use the linear eigenfunctions of Toman *et al.* (1998) to calculate the self-gravitational torque Υ. We then use Υ to study the longterm evolution of the bar mode. We compare predictions of our "quasi-linear" theory to several fully nonlinear hydrodynamics simulations. We find that: (i) the saturation of the bar mode instability is due to the self-interaction gravitational torque. Saturation occurs before fission. If the growth of the bar mode is unchecked, fission likely ensues. (ii) The quasi-linear theory accurately predicts the extent of the bar and hence the mass and other global properties of the bars. (iii) The quasi-linear theory suggests that bars will show two stages of evolution; a dynamic growth phase where the mode saturates followed by a secular phase where the bar slowly sheds angular momentum to the spiral arms.

1 Introduction

Bate (1998) recently studied the formation of low mass stars using a modern SPH code capable of resolving the protostellar collapse from interstellar to stellar densities using a piecewise polytropic EOS which mimics the transitions from isothermality to isentropic molecular gas to dissociating and ionizing gas to fully ionized gas. Bate found that when the $T/|W|$ of the first equilibrium molecular core exceeds 0.27, barlike modes grow and the extraction of angular momentum from the central regions precipitates the collapse of the first core. Here, T is the rotational kinetic energy and W is the gravitational energy of the cloud. At the same time, however, the ongoing removal of angular momentum by the bar mode prevents any fragmentation during the collapse of the first core to stellar densities. This seems to eliminate the possibility of forming close binaries during the collapse of the first core. In cases where dynamic bar mode instabilities in the first or second cores do not induce collapse, Tohline *et al.* (1998) are investigating whether the barlike remnants might evolve to close binary configurations during subsequent contraction and cooling. New & Tohline (1997) have already shown that stable common envelope configurations are possible for mildly compressible polytropes ($n \lesssim 1.5$). Given the radically different outcomes of the above scenarios, an understanding of the consequences of the bar mode instability under general conditions is needed.

2 Calculations

Recently, we studied the stability properties of the global nonaxisymmetric modes of rotating polytropes. Our precise characterization of the linear eigenmodes (Toman *et al.* 1998) allows us to predict the nature of the early nonlinear evolution of the barlike modes through a "quasi-linear" theory which computes second-order torques based on the form of the first-order eigenfunction.

2.1 Linear Analysis

We studied the secular (Imamura *et al.* 1995) and dynamic (Pickett *et al.* 1996, Toman *et al.* 1998) stability properties of Kelvin-like modes in rotating polytropes. We considered modes with $\cos(m\phi)$ dependence on the azimuthal angle ϕ and m = 1, 2, 3, and 4. We found that instability sets in through the $m = 2$ mode whenever $T/|W| > 0.26$-0.27. The eigenvalues and eigenmodes determined using our linear and nonlinear codes are in excellent agreement (Toman et al. 1998). The bar mode eigenfunction is barlike in the central regions trailing into spiral arms as the equator is approached.

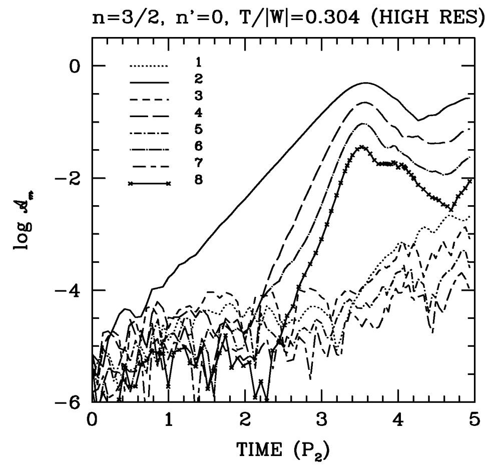

Figure 1: The time evolution of the global Fourier amplitudes $\mathcal{A}_m$ for the high resolution $(n, n') = (3/2,0)$, $T/|W| = 0.304$ nonlinear simulation. The curves show $\log(\mathcal{A}_m)$ for $m = 1$ to 8; time is given in units of the pattern period for the $m = 2$ mode, P_2.

2.2 Nonlinear Simulations

We evolved several equilibrium models predicted to be unstable by our linear analysis into the nonlinear regime using a fully 3-dimensional, hydrodynamics code. The code is a second-order, explicit, Eulerian code that solves the equations of hydrodynamics in conservative form on a uniform cylindrical grid (Pickett et al. 1996). The initial grid sizes range from (32,64,16) to (64,64,32) in cylindrical coordinates (r, φ, z). As a model expands in a nonlinear simulation, the grid can be easily enlarged radially and vertically; the maximum grid sizes used here are (256,64,16) and (256,64,32). Boundary conditions on the grid are outflow at the largest r and z and reflection through the equatorial plane. Sample output from the $(n, n') = (3/2,0)$ polytrope with $T/|W| = 0.304$ is shown in Figure 1. We show the evolution of the mode amplitudes $(\mathcal{A}_m)$. The $\mathcal{A}_m$ are Fourier amplitudes of ρ in the equatorial plane. The times are in units of the $m = 2$ mode pattern period P_2. The $m = 2$ mode grows linearly up to $t \sim 3.5\ P_2$ at which time it saturates. We show the evolution of the angular momentum contained in the bar J_b as a function of time in Figure 2 for several simulations.

2.3 Quasi-Linear Theory

A primary feature of bar mode evolution is the initial outward pulse of angular momentum transport which occurs essentially at the same time as nonlinear saturation (see Figures 1 & 2) followed by the slower decline of the bar amplitude and ejection of material spiral arms (see Figure 1 and Durisen *et al.* 1986). The torque arises because of the winding of the eigenfunction. The spiral arms trail the bar and so the gravitational torque removes J from the bar and transfers it to the arms. We use the linear eigenfunctions to calculate the torque. We refer to this as a "quasi-linear" analysis because the torques themselves are second-order in the perturbations. The torque is given by

$$\Upsilon = -\mathbf{r} \times (\delta\rho \nabla \delta\Phi) \qquad (1)$$

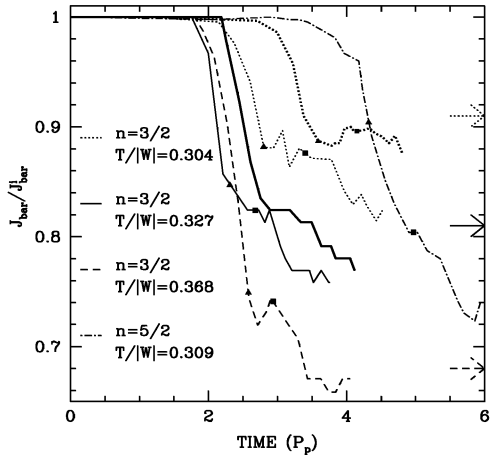

Figure 2: The time evolution of the bar angular momentum for the $(n, n') = (3/2,0)$, $T/|W| = 0.304$, 0.327, and 0.368 nonlinear simulations and the $(n, n') = (5/2,0)$, $T/|W| = 0.310$ simulation. Higher resolution simulations are represented by darker lines with the appropriate line type. The bar angular momentum J_{bar} is normalized by the initial value J^i_{bar}. A filled triangle marks the value of J_{bar}/J^i_{bar} at t_{max}; a filled square marks the value of J_{bar}/J^i_{bar} at t_{min}. The arrows at right mark the J_{dlim} for the models with the same curve type. An arrow is not shown for the $n = 5/2$ model because of inaccuracy in determination of m_{bar} and J_{bar} in the nonlinear code.

where $\delta\rho$ and $\delta\Phi$ are the Eulerian density and gravitational potential perturbations. The radial extent of the bar is determined by where Υ changes sign. The bar mass m_{bar} predicted by the quasi-linear theory agrees with nonlinear simulations to within 2 digits for all models calculated for this paper and for all models found in the literature.

The end of the bar falls well inside corotation, $R_{bar}/R_{co} \sim 0.6$ to 0.7. Lindblad resonances, when they exist, do not appear to play a significant role in the determination of the radius of the bar. We find a bar resonance which does, however, produce good predictions for the extent of the bar and hence good predictions for the global properties of the bar for all models tested. The bars extend out to the location where $\kappa/4 = \Omega_{pat}$ where κ is the epicyclic frequency and Ω_{pat} is the pattern frequency of the polytrope.

3 Discussion

Based on our linear, quasi-linear, and nonlinear results, we can imagine well-defined possibilities for the evolution of a developing bar mode. Our simulations show that the first pulse of angular momentum transport drives J_{bar} to the J of the marginally dynamically stable bar mode axisymmetric model, J_{dlim}; $J_{bar} \approx J_{dlim}$ to within 5 - 6 % at the end of the rapid transfer phase. Second, the nonlinear and quasi-linear simulations show that the self-interaction gravitational torque is the mechanism which saturates the bar mode growth. The angular momentum transfer rates predicted by the quasi-linear and nonlinear theories are in good agreement. Third, the bar is dynamically stable after the initial pulse of J transport, however, it still continues to evolve. The bar evolves at a slower rate than it did during the initial J pulse but, nonetheless, it continues to evolve. It is not the self-interaction torque of the bar mode which drives the evolution, rather, the bar likely couples to spiral modes excited in the ejected disk of material which maintains the barlike structure (Imamura et al. 1995) and drives the slow angular momentum transport from the bar to the arms. The slow, secular evolution continues until J_{bar} reaches the J for the marginally secularly stable axisymmetric model. At this point, the object becomes axisymmetric.

The slowly evolving bars suffer large central compression. For the most compressible objects we consider ($n = 5/2$ polytropes), the quasi-linear theory predicts a compression of more than 10^5 in the most extreme case. This type of behavior is shown by our nonlinear simulations.

Applied to the star formation process, we conclude that:

- The self-interaction torque saturates the growth of the bar mode saturates before it can lead to fission (cf. Durisen et al. 1986)
- The mode also saturates before strong off-center ρ maxima form. There are off-center maxima, but they are weak; relative ρ contrasts of < 10 %. The cooling scenario of Tohline et al. (1998) thus probably does not work. The density maxima will indeed grow as the bar cools, but because the bar is also rapidly compressing as it sheds J, the cooling does not have enough time to produce ρ enhancements with the amplitude necessary to produce binary stars before H_2 is dissociated and the second collapse phase is triggered. For $n = 3/2$ rather than $n = 5/2$, large central compression does not occur and cooling may be efficient enough to produce binary star systems.

4 Summary and Conclusions

We developed a quasi-linear approach for the study of the nonlinear development of dynamic bar mode instabilities in rapidly rotating polytropes. We used the linear eigenfunctions for unstable bar modes to estimate the second-order self-interaction torque between the central barlike regions, which lose angular momentum, and the outer spiral arms, which gain angular momentum. The linear eigenfunction also predicts approximate structures for the central bars. Numerical simulations confirm that the quasi-linear theory gives quantitatively accurate predictions for many aspects of the nonlinear outcome, including the bar mass, its angular momentum and vortensity distributions, and the compression of the bar material due to angular momentum loss.

Our principal result is a clear understanding of the early nonlinear evolution. The bar mode growth saturates at an amplitude such that the self-interaction torques occur on a time scale comparable to the dynamic e-folding time for the instability. The first rapid pulse of angular momentum transport generally ends once enough angular momentum is removed from the central bar regions so that, if axisymmetric, these regions would be at the barlike mode dynamic stability limit. There does appear to be a continuing drain of angular momentum after the initial pulse through further interaction between the bar and the ejected spiral arm material. This slow evolution will presumably continue until the secular bar mode stability limit is reached, unless it is interrupted by other physical effects. For example, Tohline *et al.* (1998) suggest that cooling could drive the future evolution of the bars, eventually leading to the formation of close binary star system, and Bate (1998) shows that the angular momentum loss and subsequent compression driven by development of the bar mode can trigger the dynamical collapse of the H_2 clouds by leading to the dissociation of the H_2 molecules.

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